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TIME-DEPENDENT SOLUTION OF THE INCOMPRESSIBLE NAVIER–STOKES EQUATIONS TO SIMULATE AIR-CONDITIONING

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SUMMARY

Computer simulations may contribute significantly to the optimal design of air-conditioning systems. To capture the effects of partially permeable walls such as bookshelves on the movement of air and heat, it is necessary to extend the density-dependent Navier–Stokes equations by an additional friction term. The finite element technique is convenient to approximate the extended equations in spatial co-ordinates. For the time co-ordinate a recently proposed semi-implicit finite difference method is very efficient in terms of accuracy and computational complexity. A pressure correction approach is most appropriate to decouple the primitive variables in the extended Navier–Stokes equations. The resulting algorithm has the interesting feature that small symmetric positive definite systems of equations can be solved sequentially for each of the primitive variables. Iterative solution of the systems of equations with preconditioned conjugate gradients combined with a compressed storage scheme allows fine grid computations at affordable costs. As an example a two-dimensional version of the code was applied to an enclosure which was heated from the side and contained a porous wall. The time-dependent computational results are compared with measurement data.

KEY WORDS Permeable walls Bulk friction Time-dependent Incompressible Navier–Stokes Pressure correction Projection Semi-implicit Finite elements

INTRODUCTION

Expensive and time-consuming physical models are used to estimate the optimal dimensioning of air-conditioning systems today. Recent developments in computational fluid dynamics as well as supercomputer power open new possibilities. Computer simulations may help to optimize air-conditioning systems.[1] However, the real world inside an air-conditioned building or office is frequently far from the perfect and idealized assumptions which are made by classical computational fluid dynamics codes. Especially with regard to furniture and people in a building we identified a need to develop a mathematical approach, which can be helpful to better represent a realistic situation.

THE CONCEPTUAL MODEL

Volume averaging, a simple variant of the general homogenization theory, is used to derive a mathematical tool for the simulation of air-conditioning in buildings with porous obstructions such as furniture and people. The technique has already been used earlier to verify Darcy's law for

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porous media flow[2] and to extend the viscous flow equations in regions where a porous matrix is present.[3-8] It is necessary to extend the equations in such a way that

(1) they are valid for the porous region
(2) they are valid also for the viscous region
(3) they are valid at the interfaces between the two regions
(4) they fulfil the boundary and interface conditions.

If the porous region can be characterized by a representative porosity n, which is the volume fraction of the total representative elementary volume (REV) which is occupied by the fluid, it is obvious that the conservation equations for the fluid are valid only in the pore space n. Furthermore, intrapore fluid–solid interactions have to be considered. As Irmay showed,[2] the bulk behaviour of the flow in a large enough REV of the porous region can be averaged and, by using minimum energy principles, Darcy and Forchheimer formulae can be derived. A very detailed discussion of the calculation of fluid–solid interactions for viscous flow in a prismatic microstructure was given by Du Plessis.[8] The general form of the conservation equations for mass, momentum and energy in viscous and porous regions in terms of the primitive variables, velocities v_i, $(i=1, 2, 3)$, pressure p and temperature T, is given by Daniels.[7]

The general extended conservation equations contain numerous non-linearities in the density ρ, the dynamic viscosity μ, the porosity n and the pore or fluid velocities v_i. It has been demonstrated elsewhere[7,9] that error analysis, where all non-linearities are developed into first-order Taylor series with respect to the variation in the two variables of state, pressure p and temperature T, can identify non-linearities which are important and others which are unimportant. For the anticipated applications in air-conditioning, where

(1) the variations in temperature and pressure are small
(2) energy dissipation due to viscous forces and changes in pressure are negligible
(3) the flow is basically incompressible
(4) the velocity field is divergence-free
(5) the porosity n of the porous region is a bulk constant
(6) thermal properties c_p and λ are basically constant

the conservation equations simplify substantially to

$$\frac{\partial m_i}{\partial x_i}=0, \tag{1}$$

$$\frac{\partial m_i}{\partial t}+v_j\frac{\partial m_i}{\partial x_j}=n(\rho-\rho_0)\,g_i-W_{ij}m_j-n\frac{\partial P}{\partial x_i}+\nu\frac{\partial^2 m_i}{\partial x_j^2}, \tag{2}$$

$$n\rho c_p\frac{\partial T}{\partial t}+c_p m_j\frac{\partial T}{\partial x_j}=nQ_b+\frac{\partial}{\partial x_i}\left(n\Lambda_{ij}\frac{\partial T}{\partial x_j}\right)+A_w, \tag{3}$$

$$(1-n)\rho_k c_k\frac{\partial T_k}{\partial t}=(1-n)Q_b-A_w. \tag{4}$$

Here t is the time co-ordinate, x_i are Cartesian space co-ordinates $(i=1, 2, 3)$, g_i is the vector of gravity, c_p is the fluid's thermal capacity and λ_{ij} its thermal conductivity tensor, Q_b is a volume heat source or sink, $m_i=n\rho\, v_i$ denotes the mass flux vector, ρ_0 is a reference fluid density, P is the kinematic pressure (equation (5)), W_{ij} is a tensor friction coefficient, which in the case of non-linear Forchheimer porous medium flow with the Ward[10] formula for the dimensionless

friction factor $c=0{\cdot}550$ is given by equation (6), and ν is the kinematic viscosity:

$$P=\frac{p}{\rho_0}-g_i x_i\,, \tag{5}$$

$$W_{ij}=n\,\nu k_{ij}^{-1}+n^2 c\delta_{ij}(v_k k_{kl}^{-1} v_l)^{1/2}, \tag{6}$$

where k_{ij} is the permeability tensor of the porous material and δ_{ij} is the Kronecker delta. For a prismatic microstructure Du Plessis[8] theoretically derived the following expression for the friction coefficient:

$$W_{ij}=\nu\delta_{ij}\frac{1}{d_s^2}\frac{24(1-n)^{3/2}}{[1-(1-n)^{1/2}]^2}\left(1+0{\cdot}0822\,Re\frac{1-(1-n)^{1/2}}{1-n}\right)^{1/2}, \tag{7}$$

where the porous medium Reynolds number Re indicates the ratio between linear Darcy flow and non-linear, velocity-dependent Forchheimer flow,

$$Re=\frac{n|v|}{\nu}|k|^{1/2}=\frac{n|v|}{\nu}\frac{0{\cdot}204\,n^{1/2}}{(1-n)^{3/4}}[1-(1-n)^{1/2}]\,d_s\,, \tag{8}$$

and d_s is the cross-sectional linear dimension of a porous region.[8] He was able to verify this simple analytical expression against numerical simulations by Rothmann[11] and Coulaud *et al.*,[12] who calculated the bulk force on a fluid flowing in numerically discretized voids of a model porous medium. We therefore believe that the Du Plessis formula can be used to estimate the friction tensor W_{ij} for the system configurations which are under consideration in air-conditioning problems. Λ_{ij} is the tensor thermal diffusivity of the bulk medium,

$$\Lambda_{ij}=\frac{1}{n}\lambda^n\lambda_{kij}^{1-n}+\rho c_p D_{mij}\,. \tag{9}$$

The index 'k' denotes matrix properties. D_{mij} is the mechanical dispersion tensor. For the exchange term A_w between the locally averaged fluid phase temperature T and the locally averaged solid phase temperature T_k a simple linear approximation

$$A_w=\xi(T_k-T), \tag{10}$$

with an exchange coefficient ξ which depends on the shape and size of the matrix particles, may be appropriate in most cases. Much better approximations for any type of shape functions and consideration of time-dependent temperature T_k (or concentration) gradients can be found in Reference 13. A discussion of the simplified equations (1)–(4) for conservation of mass, momentum and energy in viscous and viscous/porous flow was given in Reference 7.

PRESSURE CORRECTION SOLUTION

A new conceptual model for the flow of fluid and heat in domains with permeable obstructions is available. The conservation equations for momentum and mass are similar to those for viscous flow. Therefore it is appropriate to use a viscous flow solver. To the best of our knowledge and experience,[7] a time-dependent solution of the Navier–Stokes equations can best be obtained by pressure correction methods. Gresho[14] proposed a semi-consistent mass, semi-implicit finite element pressure correction scheme. We combined it with a stable and second/third-order-accurate time integration scheme[15] for the momentum equations[16] and used it to solve the set of

simplified equations (1)–(4). The Galerkin weak form reads

$$C^{\mathrm{T}}\mathbf{m}=g, \tag{11}$$

$$M\frac{\partial \mathbf{m}}{\partial t}+CP=f-(D+V+F)\,\mathbf{m}, \tag{12}$$

$$M_{\mathrm{s}}\frac{\partial T}{\partial t}=f_{\mathrm{s}}-(D_{\mathrm{s}}+V_{\mathrm{s}})T+A_{\mathrm{s}}(T_{\mathrm{k}}-T), \tag{13}$$

$$M_{\mathrm{k}}\frac{\partial T_{\mathrm{k}}}{\partial t}=f_{\mathrm{k}}-A_{\mathrm{k}}(T_{\mathrm{k}}-T), \tag{14}$$

where C^{T}, g, M, C, f, D, V, F, f_{s} and A_{s} are global matrices for divergence, boundary fluxes, mass, gradient, boundary traction, diffusion, convection, friction and interphase heat exchange. The shape of the matrices is arbitrary.[7] It is important to note the following, however, in the context of general quadrilateral bilinear finite elements.

1. The approximation function ϕ for velocities and temperatures is bilinear and the approximation function Ψ for the pressure is element-wise constant and equal to one or zero respectively. There is a numerical problem in the case where the porous medium friction is much larger than the viscous forces. In this case the flow field approaches potential flow. It is expensive to calculate potential flow with bilinear velocities and element-wise constant pressures.
2. The porous medium friction is split into a main diagonal part

$$F^{\mathrm{D}}_{mn}=\sum_{e=1}^{M} W^{e}_{ij}\int_{\Omega^e}\phi_m\delta_{ij}\phi^{\mathrm{T}}_n\,\mathrm{d}\Omega^e,\quad n=1,N,$$

and the rest, which is added as one of the components of the right-hand-side vector f:

$$f^{\mathrm{D}}_{m}=\sum_{e=1}^{M} W^{e}_{ij}\int_{\Omega^e}\phi_m(1-\delta_{ij})\phi^{\mathrm{T}}_n m_{jn}\,\mathrm{d}\Omega^e,\quad n=1,N.$$

DECOUPLING OF THE EQUATIONS

A most efficient time-dependent solution of the coupled system of equations (11)–(14) requires complete decoupling of the equations. The heat transport equations for the fluid and matrix phases can be decoupled from each other by the Leismann *et al.*[17] scheme. The continuity and momentum equations can be decoupled when the continuity equation is replaced by a pressure Poisson equation. The temperature and flow fields are calculated sequentially and the non-linearities in the mass fluxes m_j and buoyancy are iterated by a Picard iteration. The pressure correction algorithm which is used for the flow field is similar to the one developed by Gresho.[14] However, the Leismann and Frind scheme[15] is used to place the convective components of the momentum equation and the energy equation at the old time level. Therefore the scheme yields symmetric linear systems of equations which can be solved by the most efficient variants of the conjugate gradient method.[18] The algorithm can be summarized as follows.

0. Start-up procedure: impose the divergence-free condition $C^{\mathrm{T}}\mathbf{m}_0=g_0$ on the initial estimate for the mass fluxes.

1. Given $\mathbf{m}_n$ with $C^{\mathrm{T}}\mathbf{m}_n = g_n$, P_n, T_n and T_{kn} at time co-ordinate t_n and additionally given estimates (from the last iteration step) $\mathbf{m}_s$, P_s and T_s for the values at time co-ordinate $(t_{n+1}+t_n)/2$, where $\mathbf{m}_s$ is also divergence-free.
2. First intermediate fluxes $\tilde{\mathbf{q}}$ are calculated from the Galerkin form of the momentum equation (12) using the Leismann and Frind[15] time integration scheme with a balancing tensor diffusivity matrix and a time-weighting factor θ:

$$\left[M+\Delta t\left(D+\theta D_{\mathrm{B}}+\frac{F}{2}\right)\right]\tilde{q}_i = \left[M-\Delta t\left(V+(1-\theta)D_{\mathrm{B}}+\frac{F}{2}\right)\right]m_{in}-\Delta t M M_{\mathrm{L}}^{-1}C^i P_s+\Delta t f_i, \quad i=1,2,3, \quad \text{in } \Omega, \tag{15}$$

$$D_{\mathrm{B}mn}=\sum_{e=1}^{M}\left(\frac{\Delta t}{2}\bar{v}_i\bar{v}_j\right)^e\int_{\Omega^e}\frac{\partial\phi_m}{\partial x_i}\frac{\partial\phi_n^{\mathrm{T}}}{\partial x_j}\,\mathrm{d}\Omega^e, \quad n=1,N.$$

Leismann and Frind showed that their scheme is second-order-accurate and unconditionally stable for pure convection and $\theta=\frac{1}{2}$. For $\theta=\frac{1}{3}$ they proved third-order accuracy but then there is a stability limit. The boundary conditions are the physical boundary conditions at the new time plane t_{n+1}:

$$\tilde{q}_i = m_{i,\mathrm{R}}^{n+1} \quad \text{on } \Gamma_1,$$

$$-P_n+\nu n_i C^{i,\mathrm{T}}(n_i\tilde{q}_i)=f_n^{n+1} \quad \text{and} \quad \nu n_i C^{i,\mathrm{T}}(\tau_i\tilde{q}_i)=f_\tau^{n+1} \quad \text{on } \Gamma_2.$$

M is the Galerkin mass matrix. The index L denotes mass lumping, n_i is the outward-pointing normal vector and f represents boundary tractions. Since we generally do not have better information, we often choose $f_n=-P_n$, $f_\tau=0$, which represents 'no-slip' or 'open-outflow' boundary conditions.

It is very important to note that equation (15) is solved sequentially for $\tilde{q}_x$, $\tilde{q}_y$ and $\tilde{q}_z$. Therefore the systems of equations are small and easy to solve by conjugate gradient methods. It was already mentioned that these systems of equations are symmetric positive definite.

3. Of course, $\tilde{q}_i$ are only estimates for the true mass fluxes m_{in+1}. They do not even fulfil the continuity equation (11) in general. It is fairly simple, however, to find a closely related vector field q_i, which is divergence-free. Variational principles[7] or the projection method[14] render the following correction algorithm.

   (a) Solve a Poisson equation for a vector of Lagrange multipliers φ of length equal to the number of bilinear quadrilateral elements with the divergence of $\tilde{q}$ as the right-hand side at the time level t_{n+1}:

$$C^{j,\mathrm{T}}M_{\mathrm{L}}^{-1}C^j\varphi = C^{j,\mathrm{T}}\tilde{q}_j-g_{n+1} \quad \text{in } \Omega, \quad j=1,2,3, \tag{16}$$

   for BCs

$$n_i C^{i,\mathrm{T}}\varphi=0 \quad \text{on } \Gamma_1, \qquad \varphi=-\frac{\Delta t}{2}(F_{n+1}+P_s) \quad \text{on } \Gamma_2.$$

   (b) Postprocess the divergence-free corrected flux q_i:

$$q_i=\tilde{q}_i-M_{\mathrm{L}}^{-1}C^i\varphi \quad \text{in } \Omega, \quad i=1,2,3. \tag{17}$$

4. Also, the vector of Lagrange multipliers φ is a good estimate for the error in the pressure estimate P_s, which can be updated as

$$P_s = P_s + \frac{2\varphi}{\Delta t} \quad \text{in } \Omega. \tag{18}$$

5. With solenoidal estimates q_i of the mass flux vector m_{in+1} available from steps 1–3, the temperature fields can be calculated. Using the Leismann *et al*[17] decoupled algorithm, the following steps are performed.
   (a) Solve for the fluid temperature T_{n+1} at the new time level:

$$[M_s + \Delta t(D_s + \theta D_B + A_1)]\, T_{n+1}$$
$$= \{M_s - \Delta t[V_s + (1-\theta)D_B]\}\, T_n + \Delta t(a_1 + f_{sn+1}) \quad \text{in } \Omega, \tag{19}$$

   according to BCs

$$T_{n+1} = T_R \quad \text{on } \Gamma_1, \qquad n_i \Lambda_{ij} C^{j,T} T_{n+1} = f_{sn+1} \quad \text{on } \Gamma_2.$$

   A_1 is a diagonal matrix which contains the exchange with the solid matrix phase and a_1 is the equivalent contribution from the solid phase:

$$A_1 = A_s^L [I - (M_k^L + A_k^L \Delta t)^{-1}\, \Delta t\, A_k^L],$$
$$a_1 = A_s^L (M_k^L + A_k^L \Delta t)^{-1}\, M_k^L T_{kn} + f_{kn+1}.$$

   I is the identity matrix and the index L denotes lumped matrices.
   (b) Finally, the solid phase temperatures T_{kn+1} can be postprocessed from

$$T_{kn+1} = (M_k^L + A_k^L \Delta t)^{-1} [M_k^L T_{kn} + \Delta t(f_{kn+1} + A_k^L T_{n+1})]. \tag{20}$$

6. If T_{n+1} is close enough to T_s (convergence of T is checked by calculation of the residual norm of equation (19) with T_s used as T_{n+1} *before* solution of equation (19)), a step forward in time is taken with

$$m_{in} = q_i, \quad P_n = P_s, \quad T_n = T_{n+1}, \quad T_{kn} = T_{kn+1} \quad \text{and} \quad t_n = T_{n+1},$$

   and step 1 proceeds. If there is no convergence,

$$\mathbf{m}_s = \frac{\mathbf{m}_n + \mathbf{q}}{2} \quad \text{and} \quad T_s = \frac{T_{n+1} + T_n}{2},$$

   and the next iteration is started with step 1. In general, only two iteration steps (Picard) are needed to get convergent results, because the non-linearities in the convection and buoyancy terms are not severe.

PRELIMINARY APPLICATION RESULTS

The simplified, extended set of equations (1)–(4) and the pressure correction solution algorithm were used to solve the Beckermann *et al.*[6] problem. A two-dimensional enclosure is heated at the left-hand side and cooled at the right-hand side. Two fluid regions alongside the vertical walls are separated by a porous wall. The Rayleigh number is $Ra = 3{\cdot}7 \times 10^6$ and the Prandtl number is $Pr = 6{\cdot}44$. Four different materials and grain sizes were used by Beckermann *et al.* Figure 1 shows contours of measured densities and our calculated temperatures 2·500 s after set-up of the heating for the four examples. At that time the calculation has attained a steady state. Figure 2(a) shows the scheme of the Beckermann *et al.* problem. Figure 2(b) shows the time-dependent development of the temperature field for Example 1. Figure 2(c) shows the stream function development for Example 1.

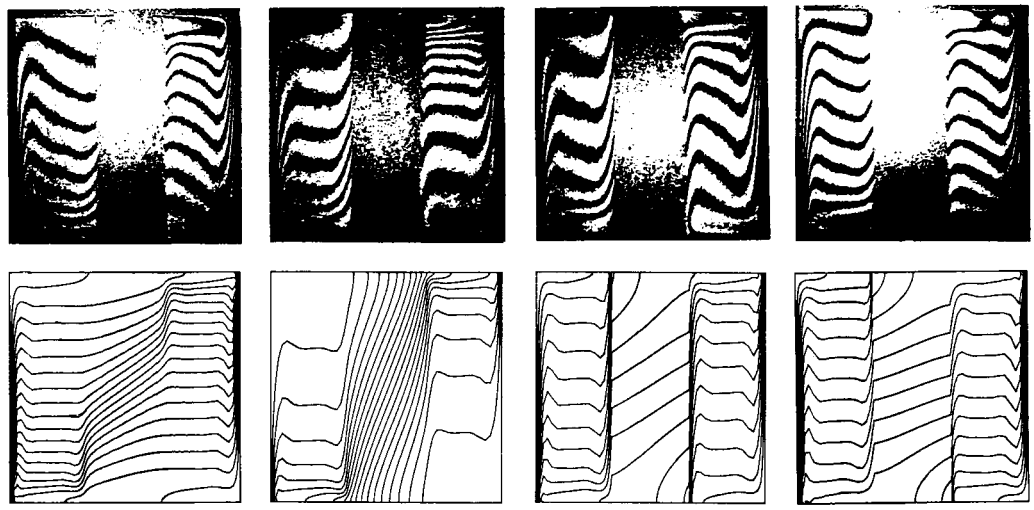

Figure 1. Mach–Zehnder interferometer measurements of Beckermann *et al.* and calculated contour plots of temperature distribution ($T_1 = 24{\cdot}2\,^{\circ}C$, $T_2 = 22{\cdot}2\,^{\circ}C$, contour line step $0{\cdot}1\,^{\circ}C$) for Examples 1–4 (from left to right)

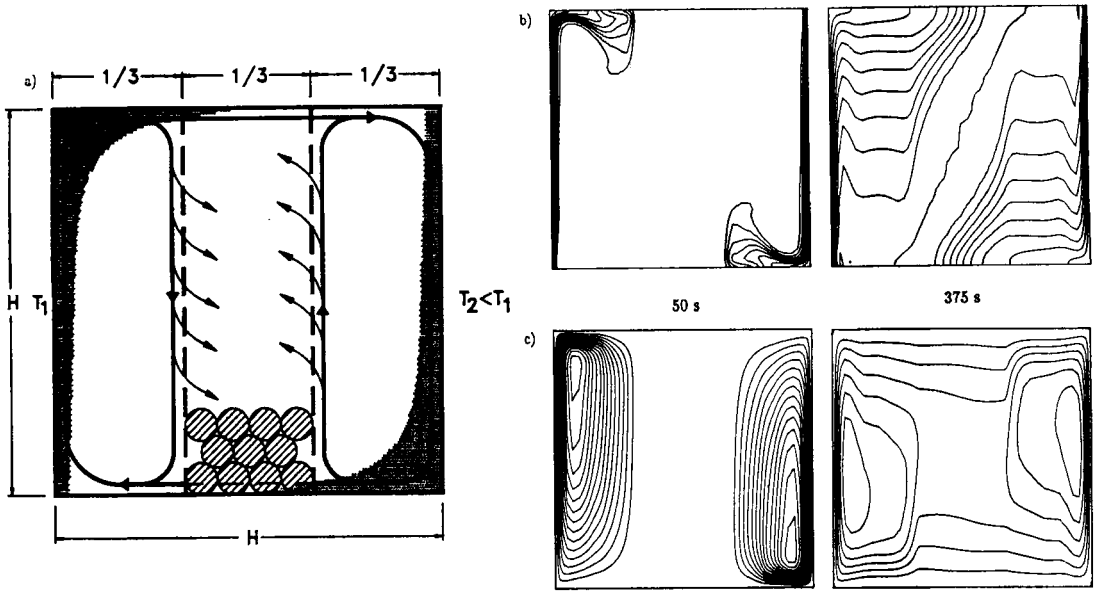

Figure 2. (a) Computational domain and time-dependent evolution of (b) temperature and (c) streamfunction for Example 1

FURTHER APPLICATIONS

Of course, the Beckermann *et al.* problem is not the ultimate goal that we are aiming at in the simulation of air-conditioning systems. However, the results which were obtained indicate that the conceptual model and the pressure correction algorithm are appropriate to simulate the flow of fluid and heat in domains with porous obstructions. With respect to air-conditioning systems we are now working on a three-dimensional version of the code and a simple eddy viscosity

turbulence model. We plan to apply the final code to the Schachenmann *et al.*[1] large-scale air-conditioning laboratory experiment and variants of it.

CONCLUSIONS

A conceptual model for the simultaneous computation of fluid and energy flow in domains with porous and permeable obstructions was proposed and discussed. The set of equations is appropriate to simulate air-conditioning problems. Since the flow equations are similar to the incompressible Navier–Stokes equations, a pressure correction algorithm combined with a semi-implicit time integration scheme for the momentum and energy equations was used to solve the Galerkin finite element algebraic equations resulting from the space–time integration of the differential equations. Preliminary application to laboratory experiments showed that the conceptual model is applicable and gives appropriate results. Furthermore, the pressure correction solution concept proved to be accurate and, to the best of our knowledge, very efficient to calculate the time-dependent development of the flow and the temperature. Since the preliminary considerations and results presented here did not show major drawbacks, we plan to apply a three-dimensional version of the code to a controlled large-scale air-conditioning experiment.

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